Handbook of Fourier Analysis

&

Its Applications

Errata

& Additional Problems

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ERRATA

- Chapter 2

    1. page 15, Table 2.3, Entry 4 for "scale then shift" should be

    $$x\left(\frac{t-\tau}{a}\right) \longleftrightarrow |a|X(au)\mathrm{e}^{-j2\pi u\tau}$$

    2. page 15, Table 2.3, Entry 5 for "shift then scale" should be

    $$x\left(\frac{t}{a}-b\right) \longleftrightarrow |a|X(au)\mathrm{e}^{-j2\pi uab}$$

Chapter 2 Problems

Review from book: Problems 1, 9(a,b,c,d,f,g,h), 10, 11, 13, 16, 17, 19, 20, 21, 23, 25, 28, 30, 31, 32, 37, 38, 41, 49, 52, 54, 55

1. Derive (2.67) using the gamma function definition in (2.64).
2. Evaluate $\int_{-\infty}^{\infty} x(t)dt$ when $x(t) =$
   (a) $(\pi t)^5 e^{-(\pi t)^2}$
   (b) $\text{sinc}(t)\ \text{sinc}^2(2t)$
   (c) $\text{sinc}(t)\ \frac{d}{dt}\text{sinc}^2(2t)$
   (d) $t^3 J_0(2\pi t)$
   (e) $\text{sinc}(2Bt)$
   (f) $\text{sinc}^2(2Bt)$
   (g) $\ln(t)\Pi\left(t-\frac{1}{2}\right)$
3. (a) Show that
   $$\Pi(t) = \Pi\left(2\left(t-\frac{1}{4}\right)\right) + \Pi\left(2\left(t+\frac{1}{4}\right)\right)$$
   (b) Since
   $$\Pi\left(2\left(t \pm \frac{1}{4}\right)\right) \longleftrightarrow \frac{1}{2}\text{sinc}\left(\frac{u}{2}\right)e^{\pm j\pi u/2}$$
   the Fourier transforming the equation in (a) looks to be
   $$\text{sinc}(u) = \text{sinc}\left(\frac{u}{2}\right)\cos\left(\frac{\pi u}{2}\right).$$
   Is this right?
4. Let $x(t)$ be a bandlimited function with maximum frequency (bandwidth) of B Hertz.
   (a) Evaluate the convolution
   $$\text{sinc}(t) * \frac{1}{2}\text{sinc}\left(\frac{t}{2}\right).$$
   (b) Show that
   $$x(t) * 2B\text{sinc}(2Bt) = x(t).$$
   (c) Is it true that, if $W \geq B$, then
   $$x(t) * 2W\text{sinc}(2Wt) = x(t)?$$
5. Simplify the following. Assume $g(t)$ is a real continuous function and that a & b are real.
   (a) $\int_{-\infty}^{\infty} g(t)\delta\left(at+b\right)dt$
   (b) $\int_{-\infty}^{\infty} g(t)\delta\left((at+b)^2\right)dt$
   (c) $\int_{-\infty}^{\infty} \Lambda(t)\delta\left(2\Lambda(t)-1\right)dt$
   (d) $\int_{-\infty}^{\infty} g(t)\delta\left(g(t)\right)dt$
6. Evaluate the convolutions
   (a) $\text{sinc}(t) * \text{sinc}(t)$
   (b) $e^{-\pi t^2} * e^{-\pi t^2}$

7. Show that
   (a) Convolution obeys
   $$\frac{d}{dt}\left(x(t) * h(t)\right) = x(t) * \frac{d}{dt}h(t) = h(t) * \frac{d}{dt}x(t)$$
   (b) If $y(t) = x(t) * h(t)$, then
   $$y(t-\tau) = x(t) * h(t-\tau) = x(t-\tau) * h(t) = x\left(t - \frac{\tau}{2}\right) * h\left(t - \frac{\tau}{2}\right) = x\left(t - \alpha\tau\right) * h\left(t - (1-\alpha)\tau\right)$$
8. Solve
   $$\sum_{n=-\infty}^{\infty} x[n]$$
   when
   (a) $x[n] = \text{sinc}\left(\frac{n}{2}\right)$
   (b) $x[n] = \sin\left(\frac{n}{2}\right)\text{sinc}\left(\frac{n}{2}\right)$
   (c) $x[n] = \text{sinc}\,(n)$
   (d) $x[n] = \Pi\,(n)$
   (e) $x[n] = n\exp(-\pi n^2)$
   (f) $x[n] = \text{sinc}^2\left(\frac{n}{2}\right)$
   (g) $x[n] = \text{sinc}^2\,(n)$
9. Evaluate the Fourier series coefficients of the periodic signals
   (a) $x(t) = e^{-\pi t^2}\sum_{n=-\infty}^{\infty} e^{2\pi n t} e^{-\pi n^2}$
   (b) $x(t) = e^{-t}\sum_{n=-\infty}^{\infty} e^{n}\mu(t-n)$
10. *Deblurring.*
    Let $M > 0$ and
    $$y(t) = x(t) * \Pi\left(\frac{t}{M}\right).$$
    Express $\frac{d}{dt}y(t)$ in terms of the sum of two shifted versions of $x(t)$.

Chapter 8 Problems

Review from book: Problems 6, 7, 12, 13, 14

1. (a) If $x\left(\vec{t}\,\right) \longleftrightarrow X\left(\vec{u}\,\right)$ when $\vec{t} = [t_1\ t_2\ t_3]^T$, find the Fourier transforms of

$$y\left(\vec{t}\,\right) = x\left(2t_2, \frac{t_3}{3}, -t_1\right)$$

and

$$z\left(\vec{t}\,\right) = x\left(t_1 + t_2, t_1 - t_2, \frac{t_3}{3}\right)$$

.

(b) Let

$$x\left(\vec{t}\,\right) = \Pi\left(t_1 - \frac{1}{2}\right)\Pi\left(\frac{t_2 - 1}{2}\right)\Pi\left(\frac{t_3 - \frac{3}{2}}{3}\right).$$

i. Sketch 3D plots of $x\left(\vec{t}\,\right)$ and $y\left(\vec{t}\,\right)$.

ii. Evaluate $X\left(\vec{u}\,\right)$, $Y\left(\vec{u}\,\right)$, and $Z\left(\vec{u}\,\right)$.

2. (a) A function is equal to one within the ellipse on the left in Figure 1 and zero outside. What is its Fourier transform?

(b) What is the Fourier transform if the function is *zero* inside the ellipse and one outside?

3. There are two circles with diameter A shown on the right in Figure 1. They are shifted on the t_1 axis as shown to be centered at $\pm a$.

(a) Assume the circles intersect, *i.e* $a < A$, and that the 2-D function is equal to zero inside of the circle's intersection. The left hand circle has a value of -1 inside of the circle that is not in the intersection. Inside the right hand circle not in the intersection, the value is 1. External to the two circles, the function is zero. Find the Fourier transform of this function.

(b) Is your result also valid if $a > A$, *i.e* the circles do don't intersect?

(c) Let's redefine the function. External to the two circles, the function is still zero. Let the value of the function inside the circle's intersection be equal to one. The rest of the function is one half. Find the Fourier transform of this function. Your result will be real.

(d) Is this result valid when the circles don't intersect?

4. A circular Dirac delta sheet can be written as

$$x(t_1, t_2) = \delta(r - r_0)$$

where, as always,

$$r = \sqrt{t_1^2 + t_2^2}.$$

Evaluate the Fourier transform of $x(t_1, t_2)$.

Chapter 11 Problems

Review from book: Problems 1, 2, 4, 5, 6, 9, 10, 11

1. Consider the following POCS problem. A real signal $x[n]$ of duration N is equal to zero outside of an interval of M consecutive points. The middle $M-2$ points in the interval are fixed whereas the endpoints in the interval are to be designed. These two points are chosen so as to best meet all of the design criteria. Let the DFT of $x[n]$ be denoted by $X[k]$. For $x[n]$ to be real, $X[k]$ must have conjugate symmetry properties. Let $X[k]$ have K points. We desire $X[k]$ to be a bandpass function over a subset of points outside of which $X[k]=0$. Within the passband, we do not care what the values are. Our job is to design a signal that meets these criteria.

   (a) Identify all applicable signal sets and show they are convex.

   (b) For each of the convex sets, provide the projection operator that projects a signal $y[n]$ onto the convex set.

   (c) Do these sets intersect?

   (d) In the POCS iteration, what is a good figure of merit (or figures of merit) by which to measure the manner in which the iteration converges?

2. Numerically explore the POCS problem described in Problem 1. This is an exploratory design problem with no set answer. We are simply asking whether the POCS approach works and, if so, under what condition?

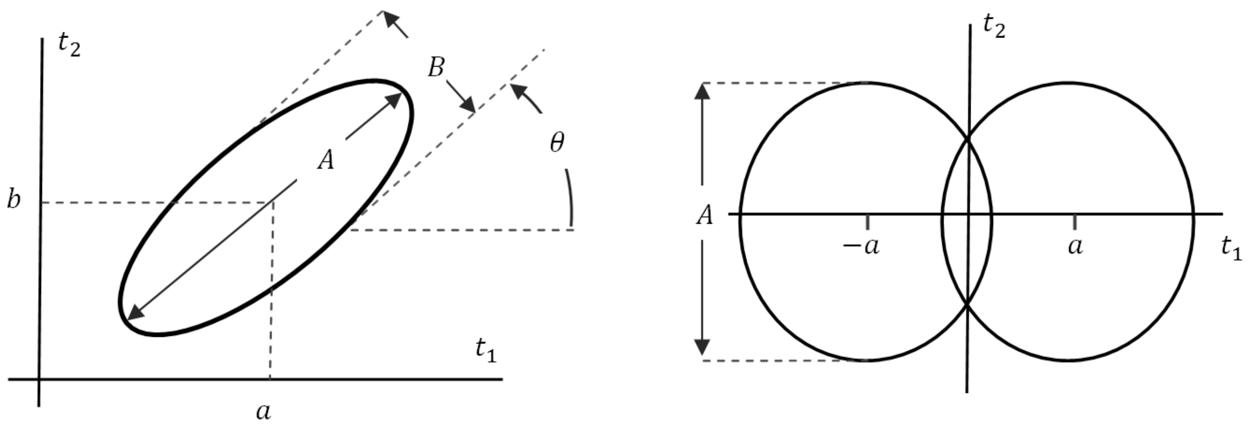

FIGURE 1. Left: A rotated and shifted ellipse. Right: Two intersecting circles.